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ON THE ROLE OF THICKNESS COMPRESSION IN SHELL DYNAMICS†

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A thin elastic shell under a transverse surface load is considered. The three-dimensional equations of the theory of elasticity are analysed asymptotically. The range of parameters of the problem is determined for which the effect of thickness compression cannot be neglected even in the roughest approximation. Copyright © 1996 Elsevier Science Ltd.

An asymptotic analysis of the three-dimensional static problem of the theory of elasticity in the case of a thin shell shows [1] that thickness compression is a second-order factor. However, this conclusion cannot always be extended to the dynamical case. For example, studies of the scattering of a plane acoustic wave by a spherical or cylindrical shell [2, 3] show that the form of partial mode resonances corresponding to a membrane wave (a symmetric null wave of Lamb type) will be severely distorted if the thickness compression of the shell by the acoustic medium is neglected. To fully understand phenomena such as those observed in [2, 3], in this paper we carry out an asymptotic analysis of the three-dimensional dynamical problem of the theory of elasticity in the case of a shell of general form with loaded face surfaces. A similar problem for a shell with free face surfaces was considered in [4].

1. FORMULATION OF THE PROBLEM

We will take the equations of motion of a shell, considered as a three-dimensional elastic body, in the form [1, 4]

$$
\begin{gathered}
L_i+\frac{1}{R\eta}a_i^{-1}\frac{\partial}{\partial\zeta}(a_i^2\tau_{i3})-\rho\left(\frac{c_s\eta^{-a}}{R}\right)^2 a_ia_j\frac{\partial^2 v_i}{\partial\tau^2}=0\\
-L+F+\frac{1}{R\eta}\frac{\partial\tau_3}{\partial\zeta}-\rho\left(\frac{c_s\eta^{-a}}{R}\right)^2 a_1a_2\frac{\partial^2 v_3}{\partial\tau^2}=0\\
Ea_je_i=a_i\tau_i-\nu a_j\tau_j-\nu\tau_3\\
\frac{E}{R\eta}a_1a_2\frac{\partial v_3}{\partial\zeta}=\tau_3-\nu a_1\tau_1-\nu a_2\tau_2\\
\frac{E}{R\eta}a_ia_j\frac{\partial v_i}{\partial\zeta}+Ea_jg_i=2(1+\nu)a_i\tau_{i3}\\
Ea_im_i+Ea_jm_j=2(1+\nu)a_j\tau_{ij}\quad(i\neq j=1,2)
\end{gathered}
\tag{1.1}
$$

Here

$$
\begin{gathered}
L_i=\frac{\eta^{-q}}{R}\left[\frac{1}{A_i}\frac{\partial\tau_i}{\partial\xi_i}+\frac{1}{A_j}\frac{\partial\tau_{ij}}{\partial\xi_j}+\eta^qk_j^*(\tau_i-\tau_j)+\eta^qk_i^*(\tau_{ij}+\tau_{ji})\right]\\
L=\frac{1}{R}\left(\frac{\tau_1}{R_1^*}+\frac{\tau_2}{R_2^*}\right)
\end{gathered}
$$

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$$F = \frac{\eta^{-q}}{R}\left(\frac{1}{A_1}\frac{\partial\tau_{13}}{\partial\xi_1} + \frac{1}{A_2}\frac{\partial\tau_{23}}{\partial\xi_2} + \eta^q k_2^* \tau_{13} + \eta^q k_1^* \tau_{23}\right)$$

$$e_i = \frac{\eta^{-q}}{R}\left(\frac{1}{A_i}\frac{\partial v_i}{\partial\xi_i} + \eta^q k_i^* v_j + \eta^q \frac{v_3}{R_i^*}\right)$$

$$m_i = \frac{\eta^{-q}}{R}\left(\frac{1}{A_j}\frac{\partial v_i}{\partial\xi_j} - \eta^q k_j^* v_j\right) \tag{1.2}$$

$$g_i = \frac{\eta^{-q}}{R}\left(\frac{1}{A_i}\frac{\partial v_3}{\partial\xi_i} - \eta^q \frac{v_i}{R_i^*}\right)$$

$$\xi_{i0} = \eta^q \xi_i, \quad R_i^* = R_i / R, \quad c_s = \sqrt{E/\rho}, \quad \eta = h/R$$

$$\tau_i = a_j \sigma_{ii}, \quad \tau_{ij} = a_i \sigma_{ij}, \quad \tau_{i3} = \tau_{3i} = a_j \sigma_{i3}, \quad \tau_3 = a_1 a_2 \sigma_{33}$$

$$a_i = 1 + \frac{\eta}{R_i^*}\zeta, \quad k_i^* = R k_i = \frac{1}{A_i A_j}\frac{\partial A_i}{\partial \xi_{j0}}$$

where σ_{kl} $(k, l = 1, 2, 3)$ are the three-dimensional stresses, v_m $(m = 1, 2, 3)$ are the three-dimensional displacements, h is the half-thickness of the shell, R is the characteristic radius of curvature of its median surface, A_i and R_i are the coefficients of the first quadratic form and the principal radii of curvature of the median surface, k_i are the geodesic curvatures of the coordinate lines of the median surface, E is Young's modulus, ν is Poisson's ratio, ρ is the density, q is the variability index, and a is the dynamics index.

The dimensionless variables ξ_i, ζ and τ are related to their dimensional counterparts α_k and t (α_i are the parameters of the curvature lines on the median surface, α_3 is the distance to the median surface measured in the normal direction, and t is the time) by the following scaling relations

$$\alpha_i = R\eta^q \xi_i, \quad \alpha_3 = R\eta\zeta, \quad t = Rc_s^{-1}\eta^a \tau \tag{1.3}$$

It is assumed that the relative half-thickness η of the shell is small and differentiation with respect to the dimensionless variables does not affect the asymptotic order of the required quantities. In addition, the inequalities

$$q < 1, a < 1 \tag{1.4}$$

which are usual in two-dimensional shell-theory [4], are imposed on the variability index and the dynamics index.

We will consider the case when the variability index and the dynamics index are related by the formula

$$q = a \tag{1.5}$$

and normal stresses $\mp q_3^\pm$ are applied to the face surfaces $\zeta = \pm 1$ $(\alpha_3 = \pm h)$ of the shell, i.e.

$$\tau_{3i}\Big|_{\zeta=\pm 1} = 0, \left.\frac{\tau_3}{a_1 a_2}\right|_{\zeta=\pm 1} = \mp q_3^\pm \tag{1.6}$$

To fix our ideas, we shall assume that $|q_3^+ - q_3^-|$ and $|q_3^+ - q_3^-|$ are commensurable. The last condition corresponds to a very general situation including, in particular, the case of a shell loaded on one of its face surfaces.

When there is no load on the face surfaces, Eq. (1.5) corresponds to moment-free vibrations ($v_i \sim v_3$) when $q = 0$, and tangential (or planar) vibrations ($v_i \gg v_3$) when $q > 0$ [4]. When the face surfaces are under load, this equality means that forced vibrations are considered for which the relationship between the frequency and the wavelength is the same as for free vibrations.

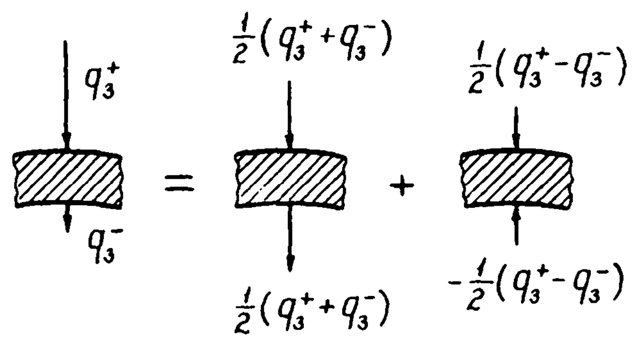

Fig. 1.

In the case of a transverse load there are two different mechanisms for exciting vibrations described by condition (1.5): omnidirectional compression and thickness compression of the shell by the load. In symbolic form (see Fig. 1) the first term on the right-hand side corresponds to omnidirectional compression and the second one to thickness compression. A priori one can expect that the effect of thickness compression will be negligible at fairly low frequencies, when the stress–strain state (SSS) of the shell is close to the static moment-free state [1]. As the frequency increases, when the strain wavelength becomes much shorter than the characteristic linear dimension of the shell, the role of the curvature of the median surface becomes secondary and the shell degenerates into a plate in the well-known sense. At such frequencies the effect of omnidirectional compression becomes negligible. Indeed, in a plate under a transverse load, vibrations corresponding to condition (1.5) (vibrations in the plane of the plate) are generated due to thickness compression alone (see, for example, [5]).

It is obvious that omnidirectional compression and thickness compression may turn out to have opposite effects. Thus, in the example shown in the figure omnidirectional compression shrinks the median surface of the shell and, conversely, thickness compression extends it. Furthermore, it can be shown that omnidirectional compression predominates when $q = a < 1/2$, thickness compression predominates when $q = a > 1/2$, and they may compensate one another when $q = a = 1/2$.

2. ASYMPTOTIC INTEGRATION

We shall take the asymptotic form of the SSS of the shell in the form

$$\begin{aligned} &v_i = R(\eta^{q-b} v_i^e + \eta^q v_i^o), \quad v_3 = R(\eta^{1-b} v_3^o + \eta^{2q-1} v_3^e) \\ &\tau_i = E(\eta^{-b}\tau_i^e + \tau_i^o), \quad \tau_{ij} = E(\eta^{-b}\tau_{ij}^e + \tau_{ij}^o) \\ &\tau_{i3} = E(\eta^{3-3q-b}\tau_{i3}^o + \eta^{1-q}\tau_{i3}^e), \quad \tau_3 = E(\tau_3^e + \tau_3^o) \quad (i \neq j = 1,2) \\ &b = 1-2q \quad (q \leq 1/2), \quad b = 0 \quad (q \geq 1/2) \end{aligned} \tag{2.1}$$

Here it is assumed that the quantities with superscripts e and o have the same asymptotic order, the quantities with superscript e being even and those with superscript o being odd functions of ζ. The set of variables $S^s = \{v_i^e, v_3^o, \tau_i^e, \tau_{ij}^e, \tau_{i3}^o, \tau_3^e\}$ and $S^a = \{v_i^o, v_3^e, \tau_i^o, \tau_{ij}^o, \tau_{i3}^e, \tau_3^o\}$ define, respectively, the symmetric and antisymmetric SSS with respect to the median surface of the shell.

The asymptotic form (2.1) corresponds to vibrations such that $v_3 \gg v_i$ when $q > 0$, unlike the similar expression in the case of free vibrations [4].

Substituting (2.1) into (1.2), we obtain

$$\begin{aligned} &L_i = \frac{E}{R}(\eta^{-q-b} L_i^e + \eta^{-q} L_i^o), \quad L = \frac{E}{R}(\eta^{-b} L^e + L^o) \\ &F = \frac{E}{R}(\eta^{3-4q-b} F^o + \eta^{1-2q} F^e), \quad e_i = \eta^{-b} e_i^e + e_i^o \\ &m_i = \eta^{-b} m_i^e + m_i^o, \quad g_i = \eta^{1-q-b} g_i^o + \eta^{q-1} g_i^e \end{aligned} \tag{2.2}$$

Here

$$
\begin{aligned}
&L_i^p = \frac{1}{A_i}\frac{\partial \tau_i^p}{\partial \xi_i} + \frac{1}{A_j}\frac{\partial \tau_{ij}^p}{\partial \xi_j} + \eta^q k_j^*(\tau_i^p - \tau_j^p) + \eta^q k_i^*(\tau_{ij}^p + \tau_{ji}^p), \qquad L^p = \frac{\tau_1^p}{R_1^*} + \frac{\tau_2^p}{R_2^*} \\
&F^p = \frac{1}{A_1}\frac{\partial \tau_{13}^p}{\partial \xi_1} + \frac{1}{A_2}\frac{\partial \tau_{23}^p}{\partial \xi_2} + \eta^q k_2^* \tau_{13}^p + \eta^q k_1^* \tau_{23}^p, \qquad e_i^p = \frac{1}{A_i}\frac{\partial v_i^p}{\partial \xi_i} + \eta^q k_i^* v_j^p + \eta^{\beta^p}\frac{v_3^p}{R_i^*} \\
&m_i^p = \frac{1}{A_j}\frac{\partial v_i^p}{\partial \xi_j} - \eta^q k_j^* v_j^p, \qquad g_i^p = \frac{1}{A_i}\frac{\partial v_3^p}{\partial \xi_i} - \eta^{\gamma^p}\frac{v_i^p}{R_i^*} \\
&\beta^e = \gamma^o = 2q - 1 + b, \quad \beta^o = \gamma^e = 1 - b
\end{aligned}
\tag{2.3}
$$

where p can take values e or o.

Now we substitute (2.1) and (2.2) into the original system of equations (1.1). Taking into account that the variables in (2.1) and (2.2) are odd or even, neglecting terms of order $\varepsilon = O(\eta + \eta^{2-2q})$ compared to unity, and using (2.3), we arrive at a closed system of equations for v_i^e, $\tau^e{}_i$, τ_{ij}^e, τ_3^e from the set S^s and also for v_3^e and τ_3^o from S^α. It has the form

$$
\begin{aligned}
&L_i^e - \frac{\partial^2 v_i^e}{\partial \tau^2} = 0, \quad -\eta^{1-b} L^e + \frac{\partial \tau_3^o}{\partial \zeta} - \frac{\partial^2 v_3^e}{\partial \tau^2} = 0 \\
&\frac{\partial \tau_3^e}{\partial \zeta} = 0, \quad \frac{\partial v_3^e}{\partial \zeta} = 0, \quad \frac{\partial v_i^e}{\partial \zeta} = 0 \\
&e_i^e = \tau_i^e - \nu \tau_j^e - \nu \eta^b \tau_3^e, \quad m_i^e + m_j^e = 2(1+\nu)\tau_{ij}^e
\end{aligned}
\tag{2.4}
$$

The remaining unknowns can be expressed in terms of the variables in (2.4) by the system of equations

$$
\begin{aligned}
&L_i^o + \frac{\partial \tau_{i3}^e}{\partial \zeta} - \frac{\partial^2 v_i^o}{\partial \tau^2} - \eta^{1-b}\zeta\left(\frac{1}{R_1^*} + \frac{1}{R_2^*}\right)\frac{\partial^2 v_i^e}{\partial \tau^2} = 0 \\
&e_i^o = \tau_i^o - \nu \tau_j^o - \nu \tau_3^o + \eta^{1-b}\zeta\left(-\frac{1}{R_j^*} e_i^e + \frac{1}{R_i^*}\tau_i^e - \frac{\nu}{R_j^*}\tau_j^e\right) \\
&\frac{\partial v_3^o}{\partial \zeta} = \eta^b \tau_3^e - \nu \tau_1^e - \nu \tau_2^e, \quad \frac{\partial v_i^o}{\partial \zeta} + g_i^e = 0 \\
&m_i^o + m_j^o = 2(1+\nu)\tau_{ij}^o + \eta^{1-b}\zeta\left[-\frac{1}{R_i^*} m_i^e - \frac{1}{R_j^*} m_j^e + \frac{2(1+\nu)}{R_j^*}\tau_{ij}^e\right]
\end{aligned}
\tag{2.5}
$$

The term τ_{i3}^o, which is asymptotically of order two, does not appear in (2.4) or (2.5). To determine this term we must refine (2.4) by introducing terms which are asymptotically of order two. However, we shall not consider this here.

Using (2.1), we rewrite the boundary conditions on the face surfaces to within an error ε as follows:

$$
\begin{aligned}
&\tau_{i3}^e\Big|_{\zeta=\pm 1} = \tau_{i3}^o\Big|_{\zeta=\pm 1} = 0 \\
&\tau_3^e\Big|_{\zeta=\pm 1} = \frac{1}{2E}(-q_3^+ + q_3^-), \quad \tau_3^o\Big|_{\zeta=\pm 1} = \mp\frac{1}{2E}(q_3^+ + q_3^-)
\end{aligned}
\tag{2.6}
$$

We now integrate over the thickness of the shell. The integral of the system of equations (2.4) satisfying boundary conditions (2.6) can be represented in the form

$$v_i^e = v_{i,0},\quad v_3^e = v_{3,0},\quad \tau_i^e = \tau_{i,0},\quad \tau_{ij}^e = \tau_{ij,0},\quad \tau_3^e = \tau_{3,0} \tag{2.7}$$

$$\tau_{i3}^o = \zeta\tau_{3,1},\quad e_i^e = e_{i,0},\quad m_i^e = m_{i,0},\quad g_i^e = g_{i,0},\quad L_i^e = L_{i,0},\quad L^e = L_{,0}$$

The functions with a comma in the subscript are independent of the transverse coordinate ζ and are related by the equations

$$e_{i,0} = \frac{1}{A_i}\frac{\partial v_{i,0}}{\partial \xi_i} + \eta^q k_i^* v_{j,0} + \eta^{2q-1+b}\frac{v_{3,0}}{R_i^*}$$

$$m_{i,0} = \frac{1}{A_j}\frac{\partial v_{i,0}}{\partial \xi_j} - \eta^q k_j^* v_{j,0},\quad g_{i,0} = \frac{1}{A_i}\frac{\partial v_{3,0}}{\partial \xi_i} - \eta^{1-b}\frac{v_{i,0}}{R_i^*}$$

$$\tau_{ij,0} = \frac{1}{2(1+\nu)}(m_{i,0}+m_{j,0}),\quad \tau_{3,0} = \frac{1}{2E}(-q_3^+ + q_3^-) \tag{2.8}$$

$$\tau_{i,0} = \frac{1}{1-\nu^2}(e_{i,0}+\nu e_{j,0}) + \frac{\nu}{1-\nu}\eta^b\tau_{3,0}$$

$$L_{i,0} = \frac{1}{A_i}\frac{\partial \tau_{i,0}}{\partial \xi_i} + \frac{1}{A_j}\frac{\partial \tau_{ij,0}}{\partial \xi_j} + \eta^q k_j^*(\tau_{i,0}-\tau_{j,0}) + \eta^q k_i^*(\tau_{ij,0}+\tau_{ji,0})$$

$$L_{,0} = \frac{\tau_{1,0}}{R_1^*} + \frac{\tau_{2,0}}{R_2^*},\quad \tau_{3,1} = -\frac{1}{2E}(q_3^+ + q_3^-),\quad \frac{\partial^2 v_{3,0}}{\partial \tau^2} = \tau_{3,1} - \eta^{1-b}L_{,0},\quad L_{i,0} - \frac{\partial^2 v_{i,0}}{\partial \tau^2} = 0$$

which are a system of 17 equations for the 17 unknown two-dimensional functions from (2.6).

The integral of the system of equations (2.5) satisfying boundary conditions (2.6) has the following form

$$v_i^o = \zeta v_{i,1},\quad v_3^o = \zeta v_{3,1},\quad \tau_i^o = \zeta\tau_{i,1},\quad \tau_{ij}^o = \zeta\tau_{ij,1} \tag{2.9}$$

$$\tau_{i3}^e = \tau_{i3,0} + \zeta^2\tau_{i3,2},\quad e_i^o = \zeta e_{i,1},\quad m_i^o = \zeta m_{i,1},\quad g_i^o = \zeta g_{i,1},\quad L_i^o = \zeta L_{i,1}$$

The two-dimensional functions in (2.9) satisfy the equations

$$v_{i,1} = -g_{i,0},\quad v_{3,1} = \eta^b\tau_{3,0} - \nu\tau_{1,0} - \nu\tau_{2,0}$$

$$e_{i,1} = \frac{1}{A_i}\frac{\partial v_{i,1}}{\partial \xi_i} + \eta^q k_i^* v_{j,1} + \eta^{1-b}\frac{v_{3,1}}{R_i^*} \tag{2.10}$$

$$m_{i,1} = \frac{1}{A_j}\frac{\partial v_{i,1}}{\partial \xi_j} - \eta^q k_j^* v_{j,1},\quad g_{i,1} = \frac{1}{A_i}\frac{\partial v_{3,1}}{\partial \xi_i} - \eta^{2q-1+b}\frac{v_{i,1}}{R_i^*}$$

$$\tau_{i,1} = \frac{1}{1-\nu^2}(e_{i,1}+\nu e_{j,1}) + \frac{\nu}{1-\nu}\tau_{3,1} + \eta^{1-b}\left[\frac{1}{1-\nu^2}\left(\frac{e_{i,0}}{R_j^*} + \frac{e_{j,0}}{R_i^*}\right) - \frac{\tau_{i,0}}{R_i^*}\right]$$

$$\tau_{ij,1} = \frac{1}{2(1+\nu)}(m_{i,1}+m_{j,1}) + \eta^{1-b}\left[\frac{1}{2(1+\nu)}\left(\frac{m_{i,0}}{R_i^*} + \frac{m_{j,0}}{R_j^*}\right) - \frac{\tau_{ij,0}}{R_j^*}\right]$$

$$L_{i,1}=\frac{1}{A_i}\frac{\partial\tau_{i,1}}{\partial\xi_i}+\frac{1}{A_j}\frac{\partial\tau_{ij,1}}{\partial\xi_j}+\eta^q k_j^*(\tau_{i,1}-\tau_{j,1})+\eta^q k_i^*(\tau_{ij,1}+\tau_{ji,1})$$

$$\tau_{i3,2}=-\frac{1}{2}L_{i,1}+\frac{1}{2}\frac{\partial^2 v_{i,1}}{\partial\tau^2}+\frac{1}{2}\eta^{1-b}\left(\frac{1}{R_1^*}+\frac{1}{R_2^*}\right)\frac{\partial^2 v_{i,0}}{\partial\tau^2},\qquad \tau_{i3,0}=-\tau_{i3,2}$$

It can be shown that if the variables in (2.9) with a comma in the subscript are known, then (2.10) is a system of 18 equations for 18 unknowns.

The form of (2.1) shows that τ_3 is asymptotically the main stress when $q \geqslant 1/2$ ($b = 0$). As follows from (2.7) and (2.8), its component symmetric with respect to the median surface is determined by the thickness compression (see the figure) to within the admissible error ε and can be expressed by

$$\tau_3^c=(-q_3^+ + q_3^-)/(2E) \tag{2.11}$$

It follows that when $q \geqslant 1/2$ thickness compression becomes asymptotically the main factor and cannot be neglected even in the roughest approximation.

3. THE AVERAGED EQUATIONS OF MOTION

We shall represent the two-dimensional equations obtained in Section 2 in terms of averaged characteristics (forces, displacements of the median surface, and so on). We will confine ourselves to Eqs (2.8).

First we will introduce some notation [1]: T_i are the normal forces, S_{ij} are the shear forces, u_i are the tangent displacements of the median surface, w is the deflection, and ε_i and ω are the components of the shear deformation. From (2.1), (2.7) and (2.9) we have

$$T_i=2Eh\eta^{-b}\tau_{i,0},\quad S_{ij}=2Eh\eta^{-b}\tau_{ij,0},\quad u_i=R\eta^{q-b}v_{i,0}$$
$$w=-R\eta^{2q-1}v_{3,0},\quad \varepsilon_i=\eta^{-b}e_{i,0},\quad \omega=\eta^{-b}(m_{i,0}+m_{j,0})$$

In this notation, Eqs (2.8) take the form

$$\begin{gathered}
\frac{1}{A_i}\frac{\partial T_i}{\partial\alpha_i}+\frac{1}{A_j}\frac{\partial S_{ij}}{\partial\alpha_j}+\frac{1}{A_iA_j}\frac{\partial A_j}{\partial\alpha_i}(T_i-T_j)+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}(S_{ij}+S_{ji})-2\rho h\frac{\partial^2 u_i}{\partial t^2}=0\\
\frac{T_1}{R_1}+\frac{T_2}{R_2}-2\rho h\frac{\partial^2 w}{\partial t^2}=-(q_3^+ + q_3^-)\\
T_i=\frac{2Eh}{1-\nu^2}(\varepsilon_i+\nu\varepsilon_j)+\frac{\nu h}{1-\nu}(q_3^- - q_3^+),\quad S_{ij}=\frac{Eh}{1+\nu}\omega\\
\varepsilon_i=\frac{1}{A_i}\frac{\partial u_i}{\partial\alpha_i}+\frac{1}{A_iA_j}\frac{\partial A_i}{\partial\alpha_j}u_j-\frac{w}{R_i},\\
\omega=\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\frac{u_1}{A_1}+\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\frac{u_2}{A_2}
\end{gathered} \tag{3.1}$$

Apart from inertial terms Eqs (3.1) are identical with the static equations of moment-free shell theory in which thickness compression is additionally taken into account (the term proportional to $(q_3^- - q_3^+)$ in the elasticity expressions for the normal forces T_i). However, whereas the term corresponding to thickness compression is asymptotically of order two in statics, when $q = a \geqslant 1/2$ it becomes asymptotically the main term in dynamics.

4. EXAMPLE

We will consider forced harmonic vibrations of an infinite circular cylindrical shell governed by the law $\exp(-i\Omega t)$ (Ω is the angular frequency), the shell being subject to a surface load uniformly distributed along the cylinder axis. Setting $R_1 \to \infty$, $R_2 \equiv R$, $u_2 \equiv u$, $\alpha_2 \equiv \alpha$, $u_1 = 0$, $\partial/\partial t = -i\Omega$ in (3.1), we obtain equations in terms of displacements and introduce the dimensionless coordinate and frequency

$$\xi = \eta^{-q}\alpha/R, \quad \Lambda = Rc_s^{-1}\eta^a\Omega \quad (a = q, \Lambda \sim \partial/\partial\xi \sim 1)$$

We shall assume that $q > 0$. Then, neglecting all terms that are asymptotically of order two, we obtain

$$\frac{\partial^2 u}{\partial\xi^2} + (1-\nu^2)\Lambda^2 u = \frac{R}{2E}\eta^{3q-1}\frac{\partial F}{\partial\xi} \tag{4.1}$$

$$F = \eta^{1-2q}\nu(1+\nu)(q_3^+ - q_3^-) - \Lambda^{-2}(q_3^+ + q_3^-)$$

The first term in the expression for F corresponds to thickness compression and the second term corresponds to omnidirectional compression. As was to be expected, thickness compression cannot be neglected even in the roughest approximation when $q \geqslant 1/2$.

If $r = (q_3^+ + q_3^-)/(q_3^+ - q_3^-)$ is positive and independent of α (for example, when $q_3^\pm = Q_3^\pm \cos(m\alpha/R)$) for $q = 1/2$, then the right-hand side of (4.1) vanishes ($F = 0$) at the frequency

$$\Lambda = \sqrt{r/[\nu(1+\nu)]} \tag{4.2}$$

The condition $r > 0$ leads to the constraint $|q_3^+| > |q_3^-|$, which shows that the frequency (4.2) only exists in the case when the amplitude of the load on the outer face surface of the shell exceeds the amplitude on the inner face surface.

Note that when the frequency (4.2) is identical with one of the shear vibration resonances, that resonance will not be excited. A similar phenomenon can be observed, for example, in acoustic wave scattering by shells [2, 3].

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REFERENCES

1. GOL'DENVEIZER A. L., *The Theory of Thin Elastic Shells*. Nauka, Moscow, 1976.
2. KAPLUNOV J. D., NOLDE E. V. and VEKSLER N. D., Asymptotic description of the peripheral waves in scattering of a plane acoustic wave by a spherical shell. *Acustica* **76**, 1, 10–19, 1992.
3. KAPLUNOV J. D., NOLDE E. V. and VEKSLER N. D., Asymptotic formulae for the modal resonances of peripheral waves in the scattering of an obliquely incident plane acoustic wave by a cylindrical shell. *Acustica* **80**, 1, 280–293, 1994.
4. KAPLUNOV J. D., KIRILLOVA I. V. and KOSSOVICH L. Yu., Asymptotic integration of the dynamical equations of the theory of elasticity in the case of thin shells. *Prikl. Mat. Mekh.* **57**, 1, 83–91, 1993.
5. GOL'DENVEIZER A. L., KAPLUNOV J. D. and NOLDE E. V., On Timoshenko–Reissner type theories of plates and shells. *Int. J. Solids Structures* **30**, 5, 675–694, 1993.

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